

How to Science

Part 1: Music



Let's say we want to make some music.

First, we need some kind of instrument. For quite some time now, we humans have known that one way to make some nice musical sounds is to take a string, fix it between two points, and add some tension.

Now, rocking out on one string can get kinda boring, and early musicians quickly figured out that to make things more interesting, they could add more strings to their instruments.

However, just adding more strings does not always make things sound better.

We have to make sure our strings sound good together. We can change the sound each string makes individually by changing its length or tension – and after a little experimentation, it becomes clear that the vast majority of combinations sound absolutely terrible.

However, every now and then, for a specific combination of string lengths and tensions, something...kinda magical happens. All of a sudden, the sounds of the strings, together, just make sense.

So if we want to make music that sounds any good, it's probably a good idea to first tune our instrument in such a way that our strings sound good together.

Early instrument makers and musicians solved this problem just like we did – by guessing and checking - until a couple thousand years ago when the Greek mathematician Pythagoras discovered something remarkable.



Figure 1 | Medieval Woodcut of Pythagoras Rocking Out. You can tell just how hard Pythagoras is rocking by his facial expression.

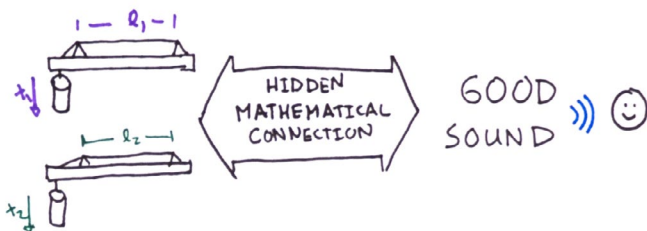


Figure 2 | Hidden Math. Pythagoras found a hidden mathematical connection between the lengths and tensions of strings that sound good together.

		STRING 1 LENGTH (cm)										
		70	65	60	55	50	45	40	35	30	25	20
STRING 2 LENGTH (cm)	70	✓	✗	✗	✗	✗	✗	✗	✓	✗	✗	✗
	65	✗	✓	✗	✗	✗	✗	✗	✗	✗	✗	✗
	60	✗	✗	✓	✗	✗	✓	✓	✗	✓	✗	✓
	55	✗	✗	✗	✓	✗	✗	✗	✗	✗	✗	✗
	50	✗	✗	✗	✗	✓	✗	✓	✗	✓	✓	✓
	45	✗	✗	✓	✗	✗	✓	✗	✗	✓	✗	✗
	40	✗	✗	✓	✗	✓	✗	✓	✗	✓	✗	✓
	35	✓	✗	✗	✗	✗	✗	✗	✓	✗	✗	✗
	30	✗	✗	✓	✗	✓	✓	✓	✗	✓	✗	✓
	25	✗	✗	✗	✗	✓	✗	✗	✗	✗	✓	✓
20	✗	✗	✓	✗	✓	✗	✓	✗	✓	✓	✓	

Figure 3 | Which String Lengths Sound Good Together? Check marks indicate which pairs of strings sound good together. You may have noticed in the video that this is a bit subjective - some string combinations sound "sortof good together" - see note 2.

Pythagoras found a hidden mathematical relationship between the length of each string, the tension in each string, and the sound the strings made together.

Let's see if we can find this hidden mathematical relationship.

We can start by making some observations.

First, we'll make tensions in each string equal, and as we change the length of each string, we'll note which combinations sound good together in Figure 3.

Next, we'll make the lengths of our two strings equal, and as we change the tension in each string, note which combinations sound good together in Figure 4.

Now that we have some data, I'm turning it over to you. Can you see what Pythagoras did?

Questions

1. What is the mathematical connection between the **lengths** of strings that sound good together? Said differently, what do the lengths of the pairs of strings that sound good together have in common?

2. What is the mathematical connection between the **tensions** of strings that sound good together? What do the tensions of the pairs of strings that sound good together have in common?

