How to Science

Part 3: Experiments

Last time, we left off wondering how the French Priest and Scientist Marin Mersenne did what Galileo said was impossible – measuring the frequency of a vibrating string.

In the early 1600s, the only really viable way to measure how frequently something was moving back and forth was…counting.

These counts could be compared to the most accurate time keeping device of the day – the pendulum. Thanks to the work of Galileo, it was known that pendulums swing back and forth at a reasonably constant frequency.[1]

So if you wanted to find the frequency of say, a vibrating string, all you really needed was a pendulum, a vibrating string, and a friend who was good at counting. After setting the string vibrating and the pendulum swinging, you could simply have your friend count string vibrations while you count pendulum swings. If you use a roughly 1 meter long pendulum, as Mersenne did, the time required for one swing is close to one second.

So if in the time it takes you to count five pendulum swings, your friend counts ten string vibrations, then each pendulum swing corresponds to two string vibrations, so your string frequency is double that of your pendulum: 2 vibrations per second.

$$\frac{16 \text{ string vibrations}}{8 \text{ pendulum swings}} = \frac{16 \text{ vibrations}}{8 \text{ seconds}} = \boxed{2 \, \frac{\text{vibrations}}{\text{second}}}$$

STRING FREQUENCY

Figure 13 | A super high-tech solution...counting. Assuming you're able to see individual string vibrations, you can measure the string's frequency by counting the number of vibrations in a given time period. The most reliable way to measure time in the 16th century was the pendulum. A one meter long pendulum takes about 1 second to complete one full swing.

In theory, this is a great way to measure the frequency of a vibrating string, but we still haven't cracked Galileo's central problem here - the vibrating string moves back and forth way too quickly for us to see.

So, how did Mersenne solve this problem?

He built a scale model.

Thanks to Galileo's educated guesses, Mersenne believed that the longer he made his vibrating strings, the lower their frequency of vibration would to be. So Mersenne set out creating longer and longer vibrating strings, until the vibrations became so slow he was able to simply count.

Mersenne's measurements were far from perfect, and he ended up with some ridiculously long strings – over 100 feet in some cases[2] - but his method worked!

He was able to measure the frequency of a vibrating string, and provide experimentally validated answers to our questions from earlier.

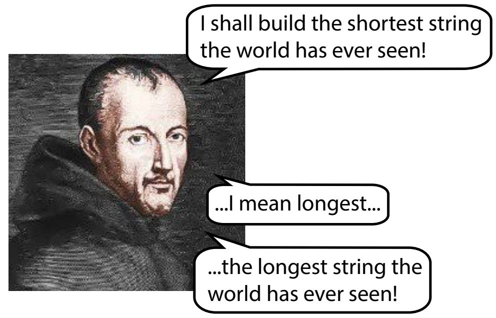

Figure 12 | Marin Mersenne. 1588-1648. French priest and scientist who did what Galileo said was impossible.

"there is no difficulty in finding the number of returns for each string proposed, for if one extends to 10 or 12 toises the length of a monochord...one can easily count its returns, the more so if it makes a very small number, for example 2 or 3 in each second. But one needs two or three [observers] to note exactly the number of these returns, one to count the returns while the other counts the number of seconds, whence if one divides the number of seconds into the number of returns, one will know how many it makes in each second."

- Marin Mersenne in Harmonie Universelle
Book 3, proposition xviii, corollary vii

Let's see if we can recreate Mersenne's approach.

Now, instead of setting up really long strings, we'll use a tool Mersenne could only have dreamed of – a high speed camera. Using our camera, we can slow down the motion of our strings – just as Mersenne did by making his strings so long – allowing us to simply count the number of vibrations in a given time period.

Figure 14 | Sony RX10 Mark IV Camera. Capable of capturing 1000 frames per second. Mersenne would have freaked out.

Now that we have a good experimental setup, we can do some real science. Let's first try to answer the first question Galileo correctly guessed the answer to:

What is the connection between the length and frequency of a vibrating string?

Now, Galileo's approach of guessing an answer first

1As long as the pendulum swings are relatively small.
2 Frederick Vinton Hunt, Origins in Acoustics, p 90.

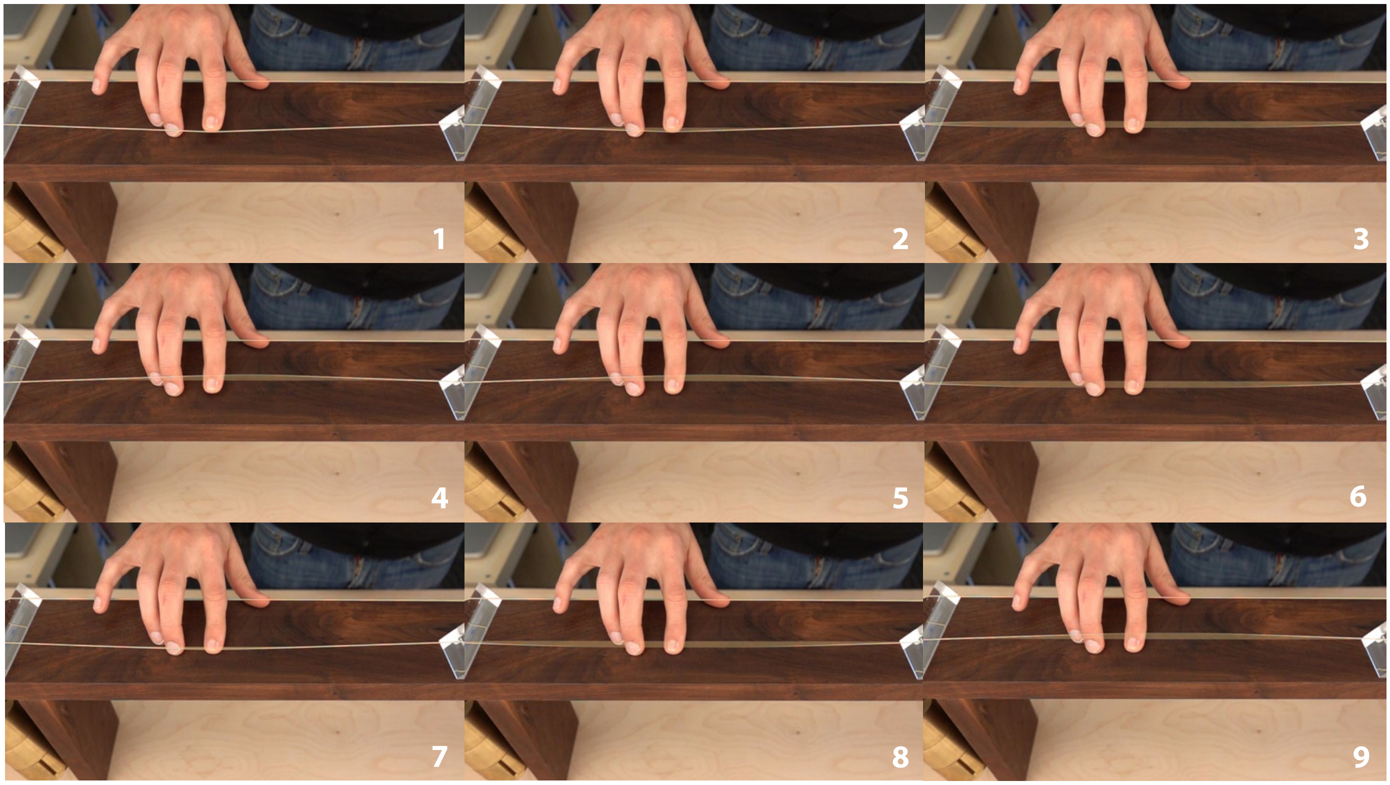

Figure 15 | Nine Consecutive Frames Captured at 480 Frames per Second. At this frame rate, it's possible to see individual string vibrations! In this string configuration (tension = 1600g, length = 40 cm), the string completes about 122 complete cycles in 1 second.

may not seem very scientific, let's see what the Nobel Prize winning physicist Richard Feynman has to say about this:

> *"Now I'm going to discuss how we would look for a new law. In general we look for a new law by the following process. First we guess it. Don't laugh, that's really true. Then we compute the consequences of the guess, and...see what it would imply. And then we compare the computation results to nature...experiment...or experience."*[3]
>
> *- Richard P. Feynman*

So, maybe Galileo's guessing first approach isn't such a bad idea. Let's start where Galileo did and see if we can make an educated guess about the connection between the length and frequency of our vibrating string.

To help us make an educated guess, let's make a few observations using the experimental setup we borrowed from Mersenne.

Let's fix our tension at 1600 g, and see how frequency changes with a few different length values. Our results are shown in Table 1. At a length of 40 cm, our string completes about 122 full cycles in 1 second, so our frequency is 122 cycles per second, also know as Hertz. If we double the length of or string, it vibrates more slowly, with a frequency of roughly 65 Hertz, and if we half our original length to 20 cm, our string vibrates more quickly, with a frequency of approximately 245 Hertz.

3 From lecture on scientific method. https://www.youtube.com/watch?v=EYPapE-3FRw

STRING 1 tension = 1600g		STRING 2 tension = 3200g	
LENGTH (cm)	FREQUENCY (Hz)	LENGTH (cm)	FREQUENCY (Hz)
80	65	80	?
40	122	50	?
20	245	40	174
		20	?

Table 1 | Can You See What Galileo Did? What is the connection between the length of a string, and it's frequency of vibration?

Now, let's do some science. Given our observations, and everything we've learned about vibrating strings – what's your guess? What do you think the connection is between the length and frequency of a vibrating string?

As Feynman pointed out, your guess should be specific enough to produce exact predictions. To make things interesting, let's go ahead and setup the experiment we'll use to test our guess. If our guess is any good, it should work for different strings and different tensions. Let's change our tension to 3200 g, and take one observation at a length of say, 40 cm. At this length and tension, or string vibrates at a frequency of 174 Hz, shown in Table 1.

Now, after we make our guess, to see if our guess is any good, we'll use it to compute what we think the frequency will be when we change this strings length to lets say, 80 cm, 50 cm, and 20 cm.

Alright, I'm turning it over to you. What's your guess? What do you think the connection is between the length and frequency of a vibrating string? Using your guess, what are your predictions for the other string lengths in our experimental setup?

Can you make the same guess that Galileo did?