How to Science

Part 6: Secret Math in Your Brain

Last time, we left off with one final question. Do the frequencies of strings that sound good together have anything in common?

Using our observations about which strings sound good together from part one, and Merenne's Laws from last time, we can compute our expected frequency for each string in our tables - results are shown in Tables 7 and 8.

And now, having a careful look at our frequencies that sound good together, we may see a hint of a pattern. If, in each case, we divide the frequency of one of our strings by the frequency of the other, our good-sounding combinations do have something interesting in common – they all reduce to simple fractions, while the ratios of the frequencies of strings that don't sound good together do not reduce to simple fractions!

This is exactly the same pattern Pythagoras discovered involving string length – the ratios of the lengths of strings that sound good together also reduced to simple fractions,

		STRING 1 LENGTH (cm)										
		70	65	60	55	50	45	40	35	30	25	20
STRING 2 LENGTH (cm)	FREQUENCY (Hz)	111.1	119.6	129.6	141.4	155.5	172.8	194.4	222.2	259.2	311.0	388.8
70	111.1	1/1	14/13	7/6	14/11	7/5	14/9	7/4	2/1	7/3	14/5	7/2
65	119.6	13/14	1/1	13/12	13/11	13/10	13/9	13/8	13/7	13/6	13/5	13/4
60	129.6	6/7	12/13	1/1	12/11	6/5	4/3	3/2	12/7	2/1	12/5	3/1
55	141.4	11/14	11/13	11/12	1/1	11/10	11/9	11/8	11/7	11/6	11/5	11/4
50	155.5	5/7	10/13	5/6	10/11	1/1	10/9	5/4	10/7	5/3	2/1	5/2
45	172.8	9/14	9/13	3/4	9/11	9/10	1/1	9/8	9/7	3/2	9/5	9/4
40	194.4	4/7	8/13	2/3	8/11	4/5	8/9	1/1	8/7	4/3	8/5	2/1
35	222.2	1/2	7/13	7/12	7/11	7/10	7/9	7/8	1/1	7/6	7/5	7/4
30	259.2	3/7	6/13	1/2	6/11	3/5	2/3	3/4	6/7	1/1	6/5	3/2
25	311.0	5/14	5/13	5/12	5/11	1/2	5/9	5/8	5/7	5/6	1/1	5/4
20	388.8	2/7	4/13	1/3	4/11	2/5	4/9	1/2	4/7	2/3	4/5	1/1

Table 7 | Connection Between Good Sound and Frequency. The frequency ratios of strings that sound good together recude to simple fractions (shown in green), while the frequency ratios of strings that don't sound good together don't!

		STRING 1 TENSION (kg)																								
		1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4
STRING 2 TENSION (kg)	FREQUENCY (Hz)	204.9	193.2	183.3	174.8	167.3	160.7	154.9	149.6	144.9	140.6	136.6	133.0	129.6	126.5	123.6	120.9	118.3	115.9	113.7	111.5	109.5	107.6	105.8	104.1	102.5
1.6	204.9	1/1	544/577	305/341	365/428	396/485	717/914	542/717	417/571	408/577	627/914	2/3	523/806	456/721	545/883	439/728	588/997	571/989	211/373	360/649	264/485	240/449	406/773	63/122	127/250	1/2
1.8	193.2	612/577	1/1	684/721	379/419	808/933	540/649	360/449	433/559	3/4	593/815	408/577	404/587	646/963	218/333	449/702	431/689	297/485	3/5	303/515	571/989	271/478	283/508	505/922	465/863	306/577
2.0	183.3	341/305	760/721	1/1	840/881	461/505	799/911	715/846	396/485	570/721	599/781	682/915	695/958	408/577	521/755	358/531	362/549	559/866	456/721	498/803	227/373	551/922	340/579	571/989	117/206	341/610
2.2	174.8	428/365	419/379	924/881	1/1	922/963	551/599	476/537	769/898	364/439	329/409	351/449	751/987	531/716	448/619	408/577	500/723	262/387	463/698	147/226	637/998	571/911	380/617	416/687	414/695	214/365
2.4	167.3	1079/881	1142/989	505/461	1010/967	1/1	955/994	649/701	305/341	808/933	799/951	396/485	662/833	433/559	542/717	387/524	437/605	408/577	415/599	214/315	2/3	218/333	559/869	456/721	275/442	297/485
2.6	160.7	914/717	780/649	911/799	599/551	994/955	1/1	927/962	755/811	585/649	829/948	781/919	761/920	803/996	572/727	113/147	727/967	315/428	468/649	408/577	671/967	571/838	545/814	314/477	147/227	457/717
2.8	154.9	1012/765	1009/809	1001/846	537/476	701/649	1017/980	1/1	940/973	869/929	873/962	239/271	588/685	461/551	396/485	631/791	575/737	333/436	336/449	419/571	692/961	408/577	107/154	166/243	502/747	506/765
3.0	149.6	571/417	559/433	1079/881	1079/924	341/305	811/755	1032/997	1/1	945/976	480/511	461/505	271/305	808/933	715/846	687/832	789/977	570/721	433/559	477/628	682/915	658/899	607/844	408/577	681/979	571/834
3.2	144.9	1393/985	4/3	912/721	439/364	1142/989	720/649	929/869	63/61	1/1	65/67	544/577	691/753	305/341	769/881	365/428	769/922	396/485	4/5	717/914	729/947	542/717	283/381	417/571	273/380	408/577
3.4	140.6	1121/769	1362/991	781/599	409/329	951/799	948/829	962/873	1055/991	67/65	1/1	69/71	612/647	886/961	341/379	407/463	190/221	829/985	489/593	714/883	634/799	367/471	722/943	475/631	391/528	664/911
3.6	136.6	3/2	1393/985	1292/963	1095/856	1079/881	919/781	271/239	505/461	612/577	71/69	1/1	73/75	684/721	649/701	379/419	882/997	808/933	605/713	540/649	396/485	360/449	583/740	433/559	381/500	3/4
3.8	133.0	1481/961	1004/691	1366/991	1309/996	1174/933	1163/962	798/685	305/271	753/691	684/647	75/73	1/1	77/79	760/799	355/382	419/461	799/898	843/967	854/999	505/602	215/261	395/488	343/431	375/479	628/815
4.0	129.6	1140/721	963/646	1393/985	716/531	559/433	996/803	551/461	1142/989	341/305	961/886	760/721	79/77	1/1	81/83	840/881	525/563	461/505	305/341	799/911	803/933	715/846	529/637	396/485	449/559	570/721
4.2	126.5	1220/753	333/218	755/521	1057/765	1012/765	924/727	1079/881	1001/846	881/769	379/341	701/649	840/799	83/81	1/1	85/87	924/967	869/929	505/551	630/701	239/271	808/933	177/208	461/551	814/989	610/753
4.4	123.6	1257/758	1207/772	531/358	1393/985	524/387	147/113	791/631	1073/886	428/365	463/407	419/379	382/355	924/881	87/85	1/1	89/91	922/963	803/856	551/599	473/524	476/537	871/1000	769/898	139/165	364/439
4.6	120.9	1075/634	1151/720	549/362	1138/787	1192/861	967/727	737/575	977/789	1151/960	221/190	1049/928	461/419	563/525	1012/967	91/89	1/1	93/95	916/955	174/185	599/649	745/822	887/996	739/844	292/339	518/611
4.8	118.3	1351/780	1277/782	1055/681	387/262	1393/985	1159/853	999/763	912/721	1079/881	985/829	1142/989	1025/912	505/461	929/869	1010/967	95/93	1/1	97/99	955/994	544/577	649/701	796/875	305/341	608/691	808/933
5.0	115.9	1393/788	5/3	1140/721	1197/794	1351/936	900/649	1049/785	559/433	5/4	593/489	713/605	1123/979	341/305	551/505	856/803	1004/963	99/97	1/1	101/103	943/980	789/835	701/755	461/505	273/304	883/999
5.2	113.7	1170/649	1562/919	1269/787	226/147	1391/945	1393/985	1003/736	628/477	914/717	1092/883	780/649	1123/960	911/799	780/701	599/551	185/174	994/955	103/101	1/1	105/107	927/962	891/941	755/811	446/487	585/649
5.4	111.5	1376/749	1351/780	1515/922	1562/997	3/2	1408/977	961/692	1292/963	947/729	799/634	1079/881	602/505	1055/908	271/239	524/473	649/599	612/577	1033/994	107/105	1/1	109/111	962/997	684/721	811/869	688/749
5.6	109.5	1738/929	1307/741	1593/952	1262/791	333/218	1155/787	1393/985	1231/901	1012/765	471/367	1009/809	261/215	1001/846	1142/989	537/476	822/745	701/649	835/789	1017/980	111/109	1/1	113/115	940/973	249/262	869/929
5.8	107.6	1883/989	1743/971	1565/919	1083/667	869/559	1395/934	1291/897	1193/858	381/283	943/722	740/583	488/395	637/529	208/177	1124/979	996/887	1019/927	755/701	941/891	997/962	115/113	1/1	117/119	914/945	555/583
6.0	105.8	1921/992	922/505	1351/780	1085/657	1140/721	477/314	1316/899	1393/985	571/417	631/475	559/433	1053/838	1079/881	551/461	1079/924	1053/922	341/305	505/461	811/755	760/721	1032/997	119/117	1/1	121/123	945/976
6.2	104.1	250/127	1520/819	206/117	695/414	1281/797	227/147	747/502	1002/697	380/273	1241/919	500/381	479/375	559/449	989/814	165/139	339/292	1024/901	304/273	487/446	869/811	262/249	1006/973	123/121	1/1	125/127
6.4	102.5	2/1	1665/883	1669/933	365/214	1277/782	1525/972	765/506	1427/977	1393/985	911/664	4/3	815/628	912/721	753/610	439/364	611/518	1142/989	999/883	720/649	749/688	929/869	583/555	63/61	127/125	1/1

Table 8 | Connection Between Good Sound and Frequency. The frequency ratios of strings that sound good together recude to simple fractions (shown in green)- while the frequency ratios of strings that don't sound good together don't!

as shown in Figure 5.

Now does our new discovery – that the ratios of the frequencies of good sounding string combinations reduce to simple fractions – get us any closer to understanding our original mystery - why these strings sound good together?

Let's return to our vibrating strings one last time, and think about how the frequency of each string affects the sound waves that reach our ears. When the ratio of our frequencies is simple, say one to two, then each time our first string completes one full vibration, our second string completes two, as shown on the left side of figure 28. This means that for each sound wave that reaches us from our first string, exactly two waves will reach us from our second string.

Now, if we change the frequency our second string, making our frequency ratio more complex – this nice pattern breaks down. The waves from our first string no longer "line up" with the waves from our second string, as shown in the right half of Figure 28.

This observation led Galileo and others to an interesting hypothesis: perhaps the sensations we feel when hearing musical sounds – our perceived consonance or dissonance – is a result of how easy it is for our ears to make sense of the patterns that reach us. Maybe our ears are just happier decoding the simpler, more predictable wave patterns that come from simple frequency ratios.

Maybe listening to music is just…counting:

And thanks to four hundred years of tremendous scientific progress since Galileo made this educated guess,

"Agreeable consonances are pairs of tones which strike the ear with a certain regularity; this regularity consists in the fact that the pulses delivered by the two tones, in the same interval of time, shall be commensurable in number, so as not to keep the ear drum in perpetual torment."

- Galileo Galilei

"Music is a hidden arithmetic exercise of the soul, which does not know that it is counting."

- Gottfried Leibniz

we've discovered that his hypothesis is…well, we actually, don't know.

While it's been observed that most people find sounds composed of simple frequency ratios more pleasant,[1] when we investigate more closely, things become significantly more complex. In the 1800s, the physician and physicist

1 Guernsey, Martha. "The role of consonance and dissonance in music." The American Journal of Psychology (1928): 173-204.
2. Malmberg, Constantine Frithiof. "The perception of consonance and dissonance." Psychological Monographs 25.2 (1918): 93.

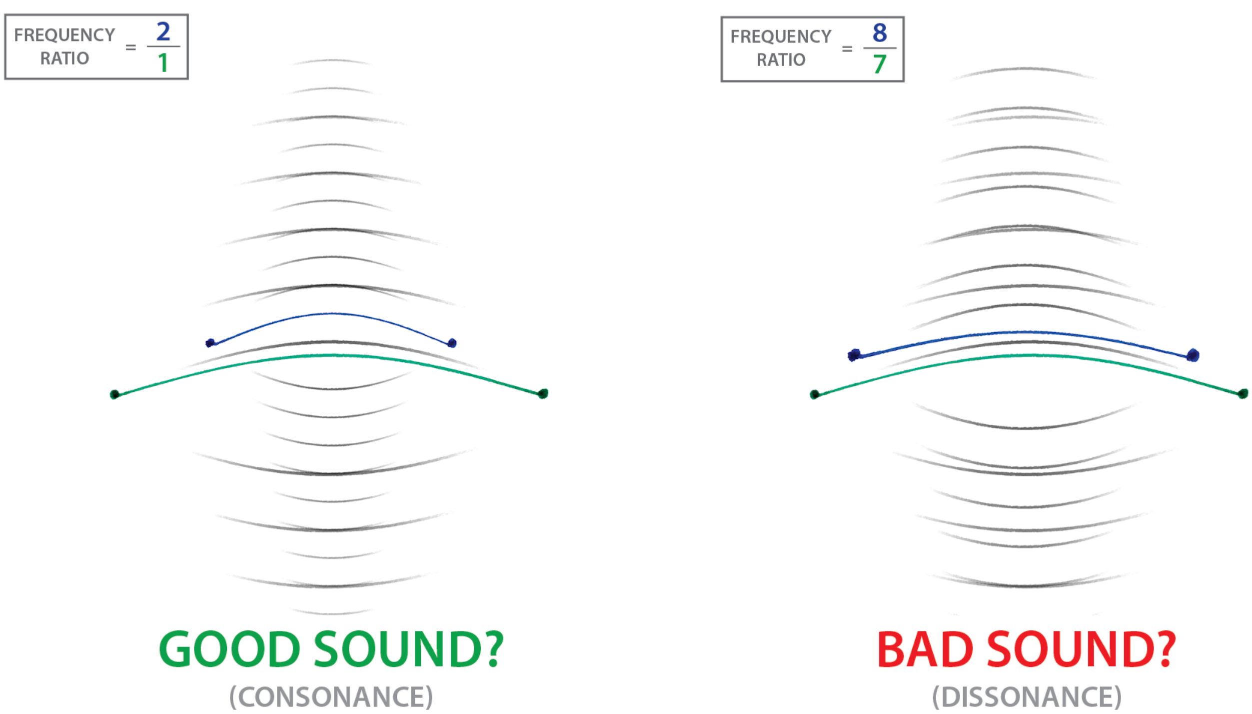

Figure 28 | Is Good Sound A Direct Result of The Wave Patterns that Reach us? Galileo and others hypothesized that simple frequency ratios sound better becuase these ratios result in simpler wave patterns.

Hermann Von Helmholtz discovered that simple frequency ratio rule breaks down when we change the nature of our sounds. It turns out that frequency ratios matter way less when listening to more pure sounds such as stopped pipe organ tones, compared to vibrating string tones.

This led Helmholtz to a new theory based on a detailed analysis of the interactions of simultaneous musical sounds. Helmholtz theory explains his observations much more effectively than the simple frequency ratio theory proposed by Galileo, however falls short in explaining a number of other observations, such as why people still prefer simple frequency ratios when tones are played one after another, instead of simultaneously. [2]

Another big complication here is that our preference for simple frequency ratios may be learned through experience. Since simple frequency ratios sounded more pleasant to Pythagoras, Galileo and to the people around them, they may have assumed that that this was true for all humans – maybe even built in to our biology.

However, in just 2016 researchers from MIT, Baylor, and Brandeis conducted a study among a group of 100 members of a remote Amazonian tribe, and found no detectable preference for simple frequency ratios.[3]

These results suggest that the reason most people find simple frequency ratios more pleasant is because we've learned to - presumably by listening to music – most of which is built on simple frequency ratios.[4]

The real complication here though is that if our preference for simple frequency ratios really is learned through experience, then the answer to the question of Pythagoras lies not in the way waves combine in the air or in the way our ears work, but instead in the 100 billion neurons our ears our attached to.

To really answer the question of Pythagoras, we may have to look inside the brain.

And beginning as early as the late 1800s , scientists began to do exactly that.

Starting with rabbits and monkeys, scientists would anesthetize the animal, surgically open up its brain, and physically place tiny electrodes to measure the electric potential across nerve fibers.[5] Scientist could then directly measure the brain's response to various sounds.

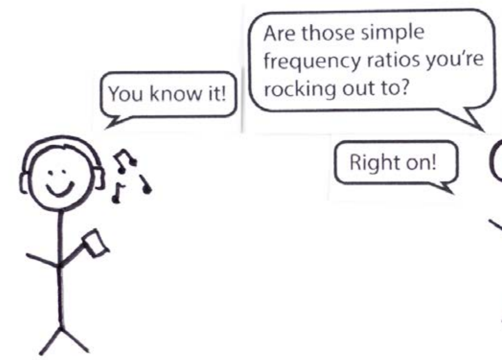

Figure 29 | Simple Frequency Ratios Rock! Most music, probably including your favorite song, is built on simple frequency ratios.

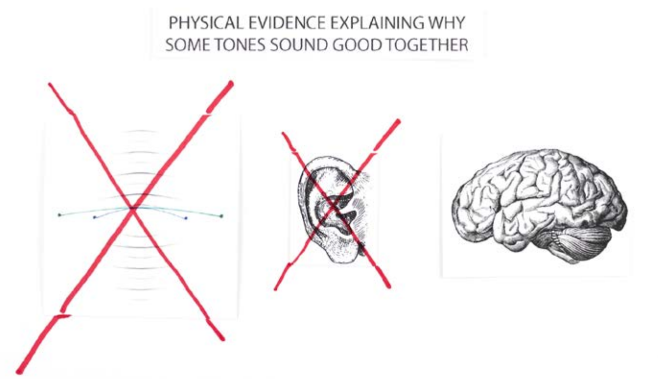

Figure 30 | Secret Math in Your Brain? We may have to look into our brains to figure out once and for all why some tones combinations sound good and others don't.

Using this technique, we've discovered some pretty incredible clues about how animal brains process sound – one remarkable discovery is that for lower frequencies [6] when an animal hears a tone, a specific nerve fiber[7] in its brain fires at the exact same frequency as the incoming tone.[8]

Now, fortunately, over the last few decades we've developed technologies over the last few decades that allows us to glimpse the inner workings of the brain without requiring brain surgery. Using electroencephalography, where electrodes are placed directly on the skull, and functional magnetic resonance imaging, where two magnetic fields and a radio frequency electromagnetic pulses are used to measure blood flow to various parts of the brain, scientists have begun to find clues about how our brains process combinations of tones.

2 Lots, Inbal Shapira, and Lewi Stone. "Perception of musical consonance and dissonance: an outcome of neural synchronization." Journal of The Royal Society Interface 5.29 (2008): 1429-1434.

3 http://news.mit.edu/2016/music-tastes-cultural-not-hardwired-brain-0713
McDermott, Josh H., et al. "Indifference to dissonance in native Amazonians reveals cultural variation in music perception." Nature 535.7613 (2016): 547.

4 DOWLING, W. J., & HARWOOD, D. L. (1986). Music cognition. San Diego: Academic Press.
LERDAHL, F., & IACKENDOFF, R. (1983). A generative theory oftonal music. Cambridge, MA: MIT Press.
SACHS, C. (1943). The rise ofmusic in the ancient world:Bast and West. New York: Norton
MEYER, L. B. (1956). Emotion and meaning in music, Chicago: University of Chicago Press.

5 https://en.wikipedia.org/wiki/Electroencephalography#History

6 Up to 2k-5k Hz

7 Or group of fibers, see Volley Theory

8 Liu, Liang-Fa, Alan R. Palmer, and Mark N. Wallace. "Phase-locked responses to pure tones in the inferior colliculus." Journal of neurophysiology 95.3 (2006): 1926-1935.
Fishman, Yonatan I., et al. "Consonance and dissonance of musical chords: neural correlates in auditory cortex of monkeys and humans." Journal of Neurophysiology 86.6 (2001): 2761-2788.
Tasaki, Ichiji. "Nerve impulses in individual auditory nerve fibers of guinea pig." Journal of Neurophysiology 17.2 (1954): 97-122.
Galambos, Robert. "Cochlear potentials elicited from bats by supersonic sounds." The Journal of the Acoustical Society of America 14.1 (1942): 41-49.

Scientist have found physical evidence for the processing of musical consonance in the brain stem[9] and cerebral cortex.[10] And some recent studies have begun mapping the brain activity of humans while listening to music,[11] one study has even begun to map brain responses across different genres of music![12]

Finally, using mathematical models of groups neurons, researchers have found evidence that our perception of consonance or dissonance may be an outcome of neural synchronization.[13]

Now, what should we do with all this information?

We've found some fascinating clues, but we have yet to find a definitive answer to the question of Pythagoras - why is it that some vibrating strings sound good together, and others don't?

Open questions like this are precisely why I love science.

We've learned great deal, but at the end of the day, no one, including the smartest people on the planet right now, can fully answer the simple question posed by Pythagoras.

And one day, someone, a person just like me or you, might just find the answer.

Thanks for reading.

9 Bidelman, Gavin M., and Ananthanarayan Krishnan. "Neural correlates of consonance, dissonance, and the hierarchy of musical pitch in the human brainstem." Journal of Neuroscience 29.42 (2009): 13165-13171

10 Itoh, Kosuke, Shugo Suwazono, and Tsutomu Nakada. "Cortical processing of musical consonance: an evoked potential study." Neuroreport 14.18 (2003): 2303-2306.

11 Zatorre, Robert J., and Carol L. Krumhansl. "Mental models and musical minds." Science 298.5601 (2002): 2138-2139.

12 Casey, Michael A. "Music of the 7Ts: Predicting and Decoding Multivoxel fMRI Responses with Acoustic, Schematic, and Categorical Music Features." Frontiers in psychology 8 (2017): 1179.

13 Lots, Inbal Shapira, and Lewi Stone. "Perception of musical consonance and dissonance: an outcome of neural synchronization." Journal of The Royal Society Interface 5.29 (2008): 1429-1434.